

# END TERM EXAMINATION

FIRST SEMESTER [MCA] DECEMBER 2013

Paper Code: MCA105

Subject: Discrete Mathematics

Time: 3 Hours

Maximum Marks: 60

**Note: Attempt any five questions including Q.no.1 which is compulsory.**

**Select one question from each unit.**

Q1. Attempt **any ten** of the following:

[10 x 2=20]

- How many diagonals are there in a regular decagon.
- Prove that  $p \rightarrow (p \vee q)$  is a tautology.
- The set  $P(\{a,b,c\})$  is partially ordered with respect to the subset relation. Find a chain of length 3 in  $P$ .
- Find the solution of recurrence relation  $a_n = 3a_{n-1} + 1$  where  $a_0 = 1$ .
- Prove that if  $\gcd(a,b) = 1$  then  $\gcd(a^2, b^2) = 1$ .
- Consider  $(m, 3m)$  encoding function, where  $m=4$ . For received word 011010011111 an error will occur or not.
- Give 2 ways to represent a graph in computer.
- Define hamiltonian graph with example.
- Show that any subgroup of a cyclic group is cyclic.
- Show that if any 5 numbers from 1 to 8 are chosen, then two of them will add up to 9.
- How many ways are there to arrange 7  $-$ sign and 5  $+$ sign, such that no two  $+$ sign are together.

## UNIT –I

Q2. a) knight is a person who always tell truth and knave always lie. We have two people A and B such that

A says “B is a Knight”, B says “the two of us are opposite”

What are A and B ? [3]

b) Let  $Z$  be the set of all integers and  $R$  be a relation defined on  $Z$  such that for any  $a, b \in Z$ ,  $aRb$  if and only if  $ab \geq 0$ . Is  $R$  an equivalence relation ? [4]

c) Show that a set of  $n$  elements can have  $2^n$  subsets. [3]

Q3.a) Prove that  $|xy| = |x||y|$  is true for all real numbers  $x$  and  $y$ . [3]

b) Define function. Find the inverse of  $f(x) = 2(x-2)^2 + 3$  for all  $x \leq 2$ . [4]

c) Find the number of integers between 1 and 100 that are divisible by any of the integer 2, 3, 5, 7. [3]

## UNIT -II

Q4.a) solve the difference equation

$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$  for  $r \geq 2$  with the boundary conditions  $a_0 = 1$  and  $a_1 = 1$ . [5]

b) let  $L_1$  be the lattice  $D_6$  (divisor of 6) = {1, 2, 3, 6} and let  $L_2$  be the lattice  $(P(S), \subseteq)$  where  $S = \{a, b\}$ . Show that two lattices are isomorphic. [5]

Q5.a) simplify  $y = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$  using K-map. [5]

b) Compute  $f(n)$  when  $n = 2^k$ , where  $f$  satisfies the recurrence relation  $f(n) = 8f(n/2) + n^2$  with  $f(1) = 1$ . [5]

## UNIT -III

Q6.a) Let  $(G, *)$  be a group. Let  $H = \{a : a \in G \text{ and } a*b = b*a \text{ for all } b \in G\}$ .

Show that  $H$  is a normal subgroup. [5]

b) Is 8792002627912 a valid universal code. Explain. [3]

c) Solve  $34x=60(\text{mod}98)$  [2]

Q7.a) A code G contains 16 code words: 0000000, 1111111, 1101000 and all its cyclic shifts, 0010111 and all its cyclic shifts. show that (G, ) is a group code. Set up the coset table to show that G can correct all single transmission errors. [5]

b) Encrypt the word 'BOOK' and 'PARK' using ceaser cipher system  
 $f(p)=p+3(\text{mod}26)$ . [5]

#### UNIT -IV

Q8.a) Define Eulerian graph. Prove that a non empty connected graph is eulerian if and only if its vertices are all of even degree. [4]

b) Differentiate between [3\*2=6]

- i. Graph and Tree
- ii. Sub graphs and isomorphic graph
- iii. Connected and complete graph

Q9.a) Prove that a planar graph G is 5 colorable. [5]

b) Explain inorder, preorder and postorder tree traversals with the help of an example. [5]