

END TERM EXAMINATION

FIRST SEMESTER [MCA] DECEMBER 2007

Paper Code : MCA107

Subject : Discrete Maths

Time : 3Hours

Maximum Marks : 60

Note: Attempt any Five question including Q.1 is compulsory. Answer one question from each unit.

- Q1.** (a) Let Z^+ be set of positive integers. Let R be a relation defined on Z^+ as follows $A R b \Leftrightarrow a$ divides b .
Give the type of relation R .
- (b) What is the generating function for the sequence 1,1,1, -----, 1
- (c) Show that $(m/r) = (m+1/r) - (m/r-1)$
- (d) Show that relation define in (i) above is a partial order relation on Z^+ .
- (e) If $S = \{1,2,3\}$ and $A = P(S)$. Is a poset with the partial orders as set inclusion? If so, draw the Hasse diagram otherwise justify your answer.
- (f) Let $B(+, ;, 1, 0, 1)$ be a Boolean Algebra. Show that, for any $a, b \in B$,
b. $(a+(a \cdot (b+b)))=b$
- (g) $(z, +)$ is a semi group z is set of integers that $(T, +)$ is a set of even integers. Show that $(T, +)$ Semigroup and define an isomorphism so that $(Z, +)$ and $(T, +)$ are isomorphic.
- (h) Let $(G, *)$ be a group. Let $a, b \in G$ then show $a * x = b$ has a unique solution in G
- (i) Define Hamilton circuit giving on example.
- (ii) What is the language generated by the following grammar
 $G = (V, S, v_0, P)$
 v_0 is start symbol, S is set of terminals, P is set of production rules
 $P = \{v_0 \rightarrow xxv_0, v_0 \rightarrow xx\}$

Unit-I

- Q2.** (a) Let R and S be relations on set A . If R and S are Symmetric then $R \cap S$ and $R \cup S$ are also Symmetric.
(b) Find the minimum number of students in a class so that of them are born in the same month.
(c) Define transitive closure of a relation R on set A . Give an example.
- Q3.** (a) Find explicit formula for the sequence define by the sequence 0,1,1,2,3,5,7,12-----.
(b) For the generating function $Z(1-Z)^{-2}$ give the generic function.

Unit-II

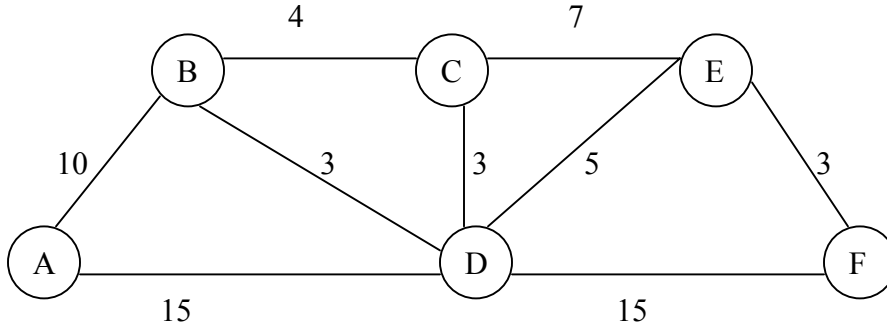
- Q4.** (a) Using Karnaugh map find the minimum sum of products from for $E = x'y + xyz' + x'y'z + xyz$
(b) Design a two-input – minimal for the Boolean expression – $abc + b'c + a'b'$
- Q5.** (a) Let (L_1, \leq) and (L_2, \geq) are Boolean Algebras, show that $(L_1 \times L_2, \leq)$ is a Boolean Algebra.

Unit-III

- Q6.** (a) Let $G = (V, E)$ be an undirected graph with e edge. Then show that $2e = \sum_{v \in V} \deg(v)$

Symbols have their own meaning. What conclusion can you draw from this result?

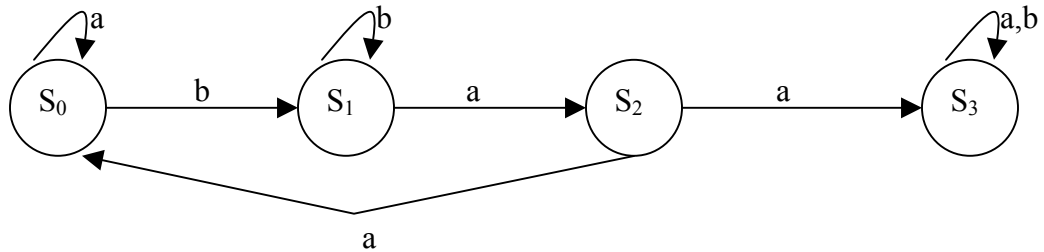
- (b) Write the steps for finding the shortest path between two vertices of a graph using Dijkstra's method hence finds the shortest path between nodes A and F



- Q.7** (a) Let (G, \cdot) be a group. Prove that $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$ for $a, b \in G$. Also Show that $((a \cdot b)^{-1})^{-1} = a \cdot b$
 (b) Let (G, \cdot) be a group. Let (H, \cdot) be a subgroup of (G, \cdot) . Show that $G = H \cup Ha \cup Hb \dots$.
 Where $a, b, \dots \in G$.

Unit – IV

- Q.8** (a) Define various phrase structured grammars. Giving an example of each.
 (b) What is the language generated by the automation?



- Q.9** (a) Draw a finite state machine accepting the language $(00)^n (11)^n 01 \quad n \geq 0$.
 (b) Let $M = (S, I, F, s_0, T)$ be a FSM
 S: Set up states
 I: Set up inputs
 s_0 : Starting State
 F is set of all transition $F: S \times I \rightarrow S$
 T is set of final states, $T \subseteq S$.
 Let R be a relation defines on S as $(s, t) \in R \Leftrightarrow s$ and t are no compatible. Show R is an Equivalence relation and R is machine congruence.