

END TERM EXAMINATION



FOURTH SEMESTER [BCA] MAY-JUNE 2012

Paper Code: BCA202

Subject: Mathematics-IV

Time : 3 Hours

Maximum Marks : 75

Note: Q.no.1 is compulsory. Attempt one question from each unit.

- Q1
- (a) In a single throw of two dice, find the probability of getting a total of 9. (2)
 - (b) Find n if $P(n,2)=72$. (2)
 - (c) Let $y=f(x)$ and $a, a+h, a+2h, \dots$ be consecutive values of x . Define the operations Δ and E as $\Delta f(x) = f(x+h) - f(x)$, $Ef(x) = f(x+h)$. Show that $E = I + \Delta$ and $E\Delta = \Delta E$. Where I denotes the identity operator. (3)
 - (d) The sum and product of the mean and variance of a binomial distribution are 24 and 128. Find the distribution. (3)
 - (e) Let X be a Poisson variate. If $P(X=0) = P(X=1)$ find $E(X)$, the expectation of X . (3)
 - (f) (i) Correlation coefficient is the Mean of the regression coefficient.
(ii) Regression coefficients are of the change of origin but of scale. (3)
 - (g) Form a forward difference table of the function $f:R \rightarrow R$ defined as $f(x) = x^3 - 4x^2 - 5x + 1$ for $x=0,1,2,3,4$; R denoting the set of all real numbers. Find $\Delta^4 f(0)$. (3)
 - (h) Prove that $\Delta(\log_e f(x)) = \log_e \left[1 + \frac{\Delta f(x)}{f(x)} \right]$ where Δ denotes the forward difference operator. (3)
 - (i) Evaluate $\int_0^4 e^x dx$ by using Simpson's one-third rule by dividing the range $[0,4]$ into 4 equal parts using $e=2.72$, $e^2=7.39$, $e^3=20.09$ and $e^4=54.6$. (3)

UNIT-I

- Q2
- (a) Let A and B be two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. Show that $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$. (6.5)
 - (b) Let A and B be two events such that $P(A) = \frac{5}{10}$ and $P(B) = \frac{8}{10}$. Find the conditional probabilities $P(A/B)$ and $P(B/A)$. (6)
- Q3
- (a) Find the number of ways in which 4 red, 3 black and 2 yellow balls can be arranged in a row. (6)
 - (b) Given $C(48,12) + C(48,13) + C(49,14) = C(50, x)$. Find x . Here $C(n,r)$ denotes the number of ways in which r objects can be chosen out of n distinct objects. (6.5)

UNIT-II

- Q4
- (a) Let X be a binomial variate with mean 4 and variance $\frac{4}{3}$. Find $P(X \geq 1)$. (6)
 - (b) Let X be a Poisson variate. If $P(X=2) = 9P(X=4) + 90P(X=6)$ show that $E(X)=1$, $E(X)$ denoting the expectation of X .

P.T.O.

Q5 (a) The lines of regression of y on x and x on y are $4x-5y+33=0$ and $20x-9y-107=0$ respectively. Determine the means \bar{x}, \bar{y} and the coefficient of correlation between x and y . (6.5)

(b) By using the normal equations, fit a parabola $Y=aX^2+bX+C$, $a \neq 0$, to the following data: (6)

x	0	1	2	3	4
y	1	5	10	22	38

UNIT-III

Q6 (a) Let $y=f(x)$ and $a, a+h, a+2h, \dots$ be the consecutive values of x . Define the operators Δ and ∇ as $\Delta f(x) = f(x+h) - f(x)$ and $\nabla f(x) = f(x) - f(x-h)$. Prove that

(i) $\Delta \nabla = \nabla \Delta = \Delta - \nabla$ (ii) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$. (6)

(b) Find a root of the equation $x^3-4x-9=0$ by using the bisection method in four stages. (6.5)

Q7 (a) Given the data:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

Find $y(1)$ and $y(10)$. (6)

(b) Given the data-

x	2	4	9	10
$f(x)$	4	56	711	980

Find the polynomial $f(x)$ by using Newton's divided difference formula. (6.5)

UNIT-IV

Q8 (a) Find the LU decomposition of the matrix $A = \begin{pmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{pmatrix}$. (6)

(b) Using Gauss Elimination method, solve the following system of linear equations. (6.5)

$$x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

Q9 (a) Using Jacobi's iteration method, solve the following system of linear equations:

$$20x + y - 2z = 17, \quad 30x + 20y - z = -18, \quad 2x - 3y + 20z = 25,$$

Hint: Start with $x_0=0, y_0=0, z_0=0$ to get the first iteration (x_1, y_1, z_1) , obtain the 5th and 6th iterations and conclude the approximate solution. (6.5)

(b) Given the data-

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\left(\frac{dy}{dx}\right)_{x=1.1}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=1.1}$ by using Newton's forward formula of interpolation. (6)