*(Please write your Exam. Roll No.)* *Exam. Roll No.* 00321402009

# END TERM EXAMINATION

## THIRD SEMESTER [BCA], DECEMBER – 2010

*Paper Code : BCA / 201* *Subject : Mathematics - III*

*Paper ID : 20201*

*Time : 3 Hours* *Maximum Marks : 75*

*Note : Q. No. 1 is compulsory. Internal choice is indicated.*

**Q. 1.** **(a)** Prove that $\sin h^{-1} x = \log(x + \sqrt{x^2+1})$ **(2.5)**

**(b)** Show that $(1+i\sqrt{3})^8 + (1+i\sqrt{3})^8 = -2^8$ **(2.5)**

**(c)** What are the Dirichlet's conditions for a Fourier series? **(2.5)**

**(d)** Prove that, for every field $\overrightarrow{V}$, $\operatorname{div}(\operatorname{curl}\overline{v}) = 0$ **(2.5)**

**(e)** Show that curl of a vector field is connected with rotational properties of the vector field and justifies the name rotation for curl. **(2.5)**

**(f)** Examine the convergence of $\sum_{n=1}^{\infty} ne^{-n^2}$ **(2.5)**

**(g)** Represent the following function by a fourier series **(2.5)**

$f(x) = x, 0 < x < 2\pi$

**(h)** Discuss the convergence and divergence of P-series. **(2.5)**

**(i)** Solve the differential equation $x^4 \frac{dy}{dx} + x^3 y = -\sec(xy)$ **(2.5)**

**(j)** Find PI of $(D^2 - 5D + 6) = e^x \cos 2x$ **(2.5)**

**Q. 2.** **(a)** Show that the function $z|z|$ is not analytic anywhere : **(3)**

**(b)** Evaluate $\lim_{z\to 1+i} \left(\frac{z-1-i}{z^2-2z+2}\right)^2$ **(3)**

**(c)** Use De Moivre's theorem to solve the equation

$x^4 - x^3 + x^2 - x + 1 = 0$ **(6.5)**

**OR**



P.T.O.

(a) Show that

$$\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right) \quad \textbf{(6.5)}$$

(b) If n is a positive integer, prove that

$$\left(\sqrt{3}+i\right)^n+\left(\sqrt{3}+i\right)^n=2^{n+1}\cos\frac{n\pi}{6} \quad \textbf{(3)}$$

(c) Write $\log(x+iy)$ in the form $a+ib$. **(3)**

**Q. 3. Solve :**

(a) $\dfrac{dy}{dx}=\dfrac{x+2y-3}{2x+y-3}$ **(6.5)**

(b) $(D^2+5D+6)\,y=e^{-2x}\sec^2 x\,(1+2\tan x)$ **(6)**

**OR**

(a) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y=e^{-x}\sec^3 x$ **(6.5)**

(b) If $\dfrac{dy}{dx}+2y\tan x=\sin x$, and $y=0$ for $x=\pi/3$, show that maximum value of y is $1/8$. **(6)**

**Q. 4.** (a) Obtain the Fourier series for function $f(x)=x^2,\ -\pi<x<\pi$, hence deduce

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$$

**(6.5)**

(b) Show that $\overrightarrow{V}=(2xy+z^3)\,i+x^2j+3xz^2k$ is a conservative field. Find its scalar potential $\phi$ such that $\overrightarrow{V}=\text{grad}\,\phi$. Find the work done by the force $\overrightarrow{V}$ in moving a particle from (1, − 2, 1) to (3, 1, 4). **(6)**

**OR**

(a) Verify the Green's theorem in the plane for

$$\int_C (3x^2 - 8y^2)\, dx + (4y - 6xy) dy$$ where C is the boundary of region

bounded by $y = \sqrt{x}, y = x^2$. **(6.5)**

(b) Represent the following function by a fourier Sine series

$$f(t) = \begin{cases} t, & 0 < t \le \dfrac{\pi}{2} \\ \dfrac{\pi}{2}, & \dfrac{\pi}{2} < t \le \pi \end{cases}$$ **(6)**

**Q. 5.** (a) Test the series $1 + \frac{2x}{\lfloor 2} + \frac{3^2 x^2}{\lfloor 3} + \frac{4^3 x^3}{\lfloor 4} + \cdots$ **(6.5)**

(b) Test the series $\frac{2}{3}x + \left(\frac{3}{4}x\right)^2 + \left(\frac{4}{5}x\right)^3 + \cdots\cdots$ **(6)**

**OR**

(a) Find the directional derivative of div $(\vec{u})$ at the point (1, 2, 2) in the direction of the outer normal of sphere $x^2 + y^2 + z^2 = 9$ for $(\vec{u}) = x^4 i + y^4 j + z^4 k$. **(3.5)**

(b) Find the value of n for which $r^n \vec{r}$ is solenoidal, $\vec{r} = xi + yj + zk$ **(3)**

(c) Test the following series for convergence and divergence

(i) Exponential Series **(2)**

(ii) Logarithmic Series **(2)**

(iii) Binomial Series **(2)**

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