(Please write your Exam Roll No.) Exam Roll No. 0421422008

END TERM EXAMINATION

THIRD SEMESTER [BCA] DECEMBER-2009

Paper Code: BCA201	Subject: Mathematics-III
Paper Id-20201	(Batch: 2006-2009)
Time : 3 Hours	Maximum Marks :75

Note: Attempt all questions with internal choice as per indicated.

Q1 (a) Find the fifth roots of (-32).

(b) Separate cosh (x+iy) into real and imaginary parts.

(c) Is the sequence $<a_n>$ defined by $a_n=0$, if n is odd, $a_n=n$, if n is even convergent?

(d) Test for convergence of the series $\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+....$

(e) Explain the convergence of logarithmic series.

(f) Give the physical interpretation of divergence or curl.

(g) If $\phi = 2x^3y^2z^4$ find $\nabla.\nabla\phi$.

(h) Is the series a $\frac{\sin x}{\sqrt{1}}+\frac{\cos 2x}{\sqrt{2}}+\frac{\sin 3x}{\sqrt{3}}+...$ a Fourier series?

(i) What is the order and degree of the differential equation $\frac{\left[1+\left(\frac{dy}{dx}\right)^3\right]^{3/2}}{\frac{d^2y}{dx^2}}=5$

(j) Find the complementary function of the differential equation $\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+8\frac{d^2y}{dx^2}-8\frac{dy}{dx}+4y=0$. **(2.5x10=25)**

UNIT-I

Q2 (a) Solve the equation $x^6+x^3+1=0$. **(6)**

(b) Test for convergence the series whose nth term is $\frac{(n!)^2}{(2n)!}x^n$. **(6.5)**

OR

(a) Prove that $-128\sin^6\theta\cos^2\theta = \cos 8\theta - 4\cos 6\theta + 4\cos 4\theta + 4\cos 2\theta - 5$. **(6)**

(b) Prove that the sequence $<S_n>$ defined by the recursion formula $S_{n+1}=\sqrt{7+S_n}$, $S_1=\sqrt{7}$ converges to the positive root of $x^2-x-7=0$. **(6.5)**

UNIT-II

Q3 (a) For what value of the constant a will the vector $A=(axy-Z^3)i+(a-z)x^2j+(1-a)xz^2k$ be irrotational. **(3)**

(b) For the function $f(x,y)=\frac{y}{x^2+y^2}$, find the value of the direction derivative making an angle 30⁰ with the positive x-axis at the point (0,1). **(3.5)**

(c) Evaluate by Green's theorem $\int_C [(\cos x\sin y - xy)dx + \sin x\cos y dy]$ where C is the circle $x^2+y^2=1$. **(6)**

OR

P.T.O.

(a) Show that gradient field describing a motion is irrotational. **(4)**

(b) If a force $F = 2x^2 y\vec{i} + 3xy\vec{j}$ displaces a particle in the xy-plane from (0,0) to (1,4) along a curve $y=4x^2$. Find the work done. **(6)**

(c) Calculate $\nabla^2\phi$ where $\phi = 4x^2 + 9y^2 + z^2 + 1$. **(2.5)**

UNIT-III

Q4 (a) Find the Fourier series of the function $f(x)=x+x^2$ in $[-1,1]$. **(6)**

(b) Show that for all values of x in $[-\pi, \pi]$ when k is not an integer, $\cos kx = \frac{\sin k\pi}{\pi}\left[\frac{1}{k} + \sum_{n=1}^{\infty}\frac{(-1)^n 2k\cos nx}{k^2 - n^2}\right]$. Deduce that $\pi\cot k\pi = \frac{1}{k} + \sum_{n=1}^{\infty}\frac{2k}{k^2 - n^2}$. **(6.5)**

OR

(a) Find the Fourier series of the function $f(x) = \begin{cases} -1, & -\pi < x < -\pi/2 \\ 0, & -\pi/2 < x < \pi/2 \\ +1, & \pi/2 < x < \pi \end{cases}$. **(6.5)**

(b) Find the Fourier half-range even expansion of the function $f(x) = \left(\frac{-x}{l}\right) + 1,\ 0 \le x \le l$. **(6)**

UNIT-IV

Q5 (a) Solve by method of undetermined coefficients $(D^2-2D+3)y=x^3+\sin x$. **(6)**

(b) Solve $\frac{d^2y}{dx^2} - 4y = x\sinh x$. **(6.5)**

OR

(a) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin\log x$. **(6.5)**

(b) Is the equation $(x^4y^4 + x^2y^2 + xy)y\,dx + (x^4y^4 - x^2y^2 + xy)x\,dy = 0$ exact? If not, reduce it to an exact equation and hence solve. **(6)**

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