(Please write your Exam Roll No.) Exam Roll No. 02914422007

END TERM EXAMINATION

THIRD SEMESTER [BCA] DECEMBER-2008

Paper Code: BCA201	Subject: Mathematics-III
Paper Id: 20201	(Batch: 2005-2007)
Time : 3 Hours	Maximum Marks :75

Note: Q.1 is compulsory. Internal choice is indicated.

Q1 (a) Prove that the argument of the product of three complex numbers is equal to the sum of their arguments.

(b) Find $\nabla|\bar{r}|^3$

(c) Find P.I. for the differential equation $(D^2-3D+2)y = 2e^x \cos\left(\frac{x}{2}\right)$.

(d) Is it possible to have Fourier expansion of the function given by $f(x)=\sin\frac{1}{x}$ in the interval $(-\pi,\pi)$. $-\pi < x < \pi$ $\sum b_n \sin nx$

(e) Prove that $\langle(-1)^n\rangle$ is not a Canchy Sequence. Is it bounded?

(f) Evaluate $\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right]$.

(g) Discuss the convergence of $\sum_n \sqrt{n}\tan\frac{1}{\sqrt{n}}$.

(h) If $\bar{v}=\bar{w}\times\bar{r}$, then prove that $\bar{w}=\frac{1}{2}curl\ \bar{v}$, where $\bar{w}$ is a constant vector.

(i) Write the Fourier expansion of sin2x.

(j) State Root test and Ratio test for series. Which of them is stronger? **(2.5x10=25)**

Q2 (a) Solve: (i) $\frac{dy}{dx}=\frac{x+y-2}{y-x-4}$ (ii) $x^4\frac{d^3y}{dx^3}+2x^3\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=1$

VPP (iii) $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2\cos^2 x$ **(3+5+4.5)**

OR

(i) $(D^2+1)y=\sin x\sin 2x$ (ii) $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$ **(6+6.5)**

Q3 (a) Find the Fourier series for the function given by $f(x)=x\sin x, -\pi<x<\pi$.

Deduce that $\frac{\pi}{4}=\frac{1}{2}+\frac{1}{1.3}-\frac{1}{3.5}+\ldots$

(b) If $f(x)=\begin{cases}\omega x, & 0\le x\le\frac{1}{2}\\ \omega(l-x), & \frac{1}{2}<x<l\end{cases}$ then show that [0, l]

$f(x)=\frac{4\omega l}{\pi^2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\sin\frac{(2n+1)\pi x}{l}$. Also, find the sum $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ **(6+6.5)**

OR

(a) Find the half-range cosine series for $(x-1)^2$, 0<x<1. Deduce that $\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$

$a_n=\frac{2}{\pi}\int$

P.T.O.

(b) If $f(x)=\begin{cases} 0 & -\pi \le x \le 0 \\ \dfrac{\pi x}{4} & 0 < x < \pi \end{cases}$, then show that

$f(x)=\dfrac{\pi^2}{16}+\sum\dfrac{\left((-1)^n-1\right)}{4n^2}\cos nx-\sum\dfrac{(-1)^n\pi}{4n}\sin nx.$ Deduce that

$\dfrac{\pi^2}{8}=1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots$ **(6+6.5)**

Q4 (a) Prove that $\nabla \times (\nabla \times \vec{a}) = -\nabla^2 \vec{a} + \nabla(\nabla . \vec{a})$. **(6)**

(b) Verify Green's theorem in the plane for $\oint_c (3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region bounded by $y=\sqrt{x}$ and $y=x^2$. **(6.5)**

OR

(a) Prove that $\nabla^2(\phi\psi)=\phi\nabla^2\psi+2\nabla\phi\nabla\psi+\psi\nabla^2\phi$. **(6)**

(b) Using Green's theorem, evaluate $\oint (x^2-2xy)dx+(x^2y+3)dy$ around the boundary of the region given by $y^2=8x$ and $x=2$. **(6.5)**

Q5 (a) Express $\sin^6\theta$ in terms of cosines of multiples of θ. **(5.5)**

(b) Show that the sequence $<x_n>$ given by $x_1=1$, $x_{n+1}=\dfrac{3+2x_n}{2+x_n}$, $n \ge 1$ is convergent to $\sqrt{3}$. **(7)**

OR

(a) If $Z=\cos\theta+i\sin\theta$, then prove that $Z^n-\dfrac{1}{Z^n}=2i\sin(n\theta)$. **(5.5)**

(b) Test the convergence of the series $\sum_n \dfrac{1}{n!}\left(\dfrac{2+n}{e}\right)^n$. **(7)**

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