(Please write your Exam Roll No. immediately) Exam Roll No. ....

END-TERM EXAMINATION

THIRD SEMESTER [BCA] DECEMBER-2007

Paper Code: BCA-201, Paper Id-201923 **Subject: Mathematics-III**
Batch: (2005-2006)

Time : 3 Hours **Maximum Marks : 75**

Note : Attempt six questions in all including questions 1 which is compulsory.

Q.1 (a) Show that the series $1 + r + r^2 + r^3 + \ldots\ldots\ldots\infty$ is convergent if $|r| < 1$. (2.5x10=25)

(b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

(c) A 3x3 real matrix has an eigen value i, then its other two eigen-values can be
(i) 0, 1 (ii) -1, i (iii) 2i, -2i (iv) 0, -i

(d) Solve $y = (x-a)\,p - p^2$

(e) Find a unit vector normal to the surface
$x^3 + 3y^2 + 2z^2 = 6$ at the point (2, 0, 1).

(f) Find the P.I of $(D^2 + 4)\,y = \sin 2x$

(g) Consider a vector space $V = \mathbb{R}^2(\mathbb{R})$. Let $W = \{(2a, a) : a \in \mathbb{R}\}$. Show that W is a subspace of V.

(h) Define Innerproduct space.

(i) Solve $(x^3 + 3xy^2)\,dx + (y^3 + 3x^2y)\,dy = 0$

(j) State Sylvester's inequality.

Q.2 (a) Show that a convergent sequence of real number is bounded. (3)

(b) Examine the convergence or divergence of the following series: (4)

(i) $\dfrac{1}{2^2} + \dfrac{2^2}{3^3} + \dfrac{3^3}{4^4} + \ldots\ldots\ldots\ldots\ldots\ldots$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

(c) State the Cauchy's integral test for convergence and examine the convergence of $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$ (3)

Q.3 (a) Define the following with examples: (4)

(i) Vector space
(ii) Linear dependence and independence of vectors
(iii) Basis and dimension
(iv) Eigen values and Eigen vectors.

(b) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (3)

Find the matrix P such that $P^{-1}AP$ is a diagonal matrix.

P.T.O.

(c) Consider the vector space $V=R^3(R)$ with the usual Euclidian Inner Product. Transform the basis $V_1=(0,1,1)$, $V_2=(0,0,1)$, $V_3=(1,1,1)$ into an orthonormal basis by using Gram-Schmidt process. (3)

Q.4 Solve the following differential equations: (any three) (3+3+4)

(a) $\frac{dy}{dx} = \sec(x+y)$

(b) $(D^2-2D+1)y = xe^x \sin x$; when $D \equiv \frac{d}{dx}$

(c) $P = \log(px - y)$

(d) $\frac{dy}{dx} + (\tan x + \frac{1}{x})y = \frac{\operatorname{Sec} x}{x}$

Q.5 (a) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$ (3)

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \tan x$ (3)

(c) Solve the following simultaneous equations (4)

$$(D+2)x + (D+1)y = 0$$
$$5x + (D+3)y = 0$$

Where $D \equiv \frac{d}{dx}$

Q.6 (a) Use the method of separation of variables and solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,o) = 6e^{-3x}$ (3)

(b) Determine the solution of separation of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(o,t) = o$, $u(l,t) = o$ $(t > o)$ and the initial condition $u(x,o) = x$; l being the length of the bar. (7)

Q.7 (a) Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$

(b) Find the directional derivative of $\phi(x,y,z) = xyz$ at the point (1, -1, 2) in the direction of the vector $(2\hat{i} - 2\hat{j} + \hat{k})$. (3)

(c) Find the divergence and curl of the vector field (4)

$$V = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$$

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