

(Please write your Exam Roll No.)

Exam Roll No. .... 2111192061

**END TERM EXAMINATION**

SECOND SEMESTER [BCA] MAY-2008

Paper Code: BCA102

Paper Id: 20102

Time : 3 Hours

Subject: Mathematics-II

(Batch: 2005-2007)

Maximum Marks :75

**Note: Q.1 is compulsory. Attempt one question from each section.**

- Q1 (a) Give all partitions of  $S=\{2,3,4\}$ . (2)
- (b) Let  $f(x)=x^2-2$ ,  $g(x)=3x$  and  $h(x)=(x+1)^2$  be functions on  $\mathbb{R}$ . Find  $goh$ ,  $f^2og$ ,  $g^3$ . (3)
- (c) Let  $A=\{2,3,7,8\}$ ,  $B=\{1,3,5\}$ ,  $C=\{3,5,9,11\}$  find (i)  $B\oplus C$  (ii)  $(A-B)\cup(B-C)$  (iii)  $(A\times B)\cap(B\times B)$ . (3)
- (d) Give an example of a relation which is (i) neither symmetric nor anti-symmetric (ii) not irreflexive. (3)
- (e) Give a topological sorting of the poset  $(D_{24}, |)$ , where  $D_n$  denotes the set of all positive divisors of  $n$  and  $|$  denote divides. (3)
- (f) Give an example of an infinite lattice with finite length. (2)
- (g) Find the angle between the line  $\frac{x-3}{2} = \frac{y-1}{4} = \frac{z-2}{3}$  and the plane  $x-y+2z=3$ . (3)
- (h) What is the shortest distance between two given lines? Also, give the equations of shortest distance. (3)
- (i) Change the order of integration in  $I = \int_0^{2a} \int_{x^2/4a}^{a-x} f(x,y) dx dy$ . (3)

**SECTION-A**

- Q2 (a) Using set theory, prove the identity  $(A\times B)\cap(P\times Q)=(A\cap P)\times(B\cap Q)$ . (6.5)
- (b) Find whether the function  $f:\mathbb{N}\rightarrow\mathbb{N}$  defined by  $f(n)=n^2+n+1$  is invertible or not. (6)
- Q3 (a) Given  $A=\{1,2,3,4,5,6\}$ . Let  $R$  be a relation on  $A$  defined as  $R=\{(x,y); x+y \text{ is a divisor of } 24\}$  (6.5)
- (i) Determine the matrix of relation  $R$ .
- (ii) Find the composition  $R\circ R$ .
- (iii) Find the domain and range of  $R$ .
- (iv) Compute transitive closure of  $R$ .
- (b) Find the domain and range of the functions (i)  $f(x) = \frac{1}{\sqrt{x-2}}$  (ii)

$$f(x) = \frac{|(x-3)|}{(x-3)}$$

(6)

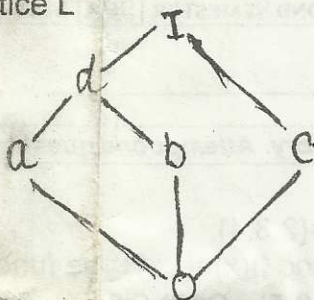
**SECTION-B**

- Q4 (a) Consider the poset  $(\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq)$  (6.5)
- (i) Find all maximal and minimal elements.
- (ii) Find the first and last elements.
- (iii) Find all the upper bounds of  $\{\{2\}, \{4\}\}$  and its supremum, if it exists.
- (iv) Find all the lower bounds of  $\{1,3,4\}$  and its infimum, if it exists.
- (b) In a distributive lattice, if an element has a complement then this complement is unique. (6)
- Q5 (a) Consider a relation  $R$  on the set  $\mathbb{Z}$  of all integers as follows  $aRb \Leftrightarrow a+b$  is even for all  $a, b \in \mathbb{Z}$ . Is  $R$  a partial order relation? Prove or give a counter example. (4)

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- (b) Give an example of a relation, on the set {1,2,3} which is both a partial ordering and an equivalence relation. (2.5)  
 (c) Consider the bounded lattice L (6)



- (i) Find all join-irreducible elements.  
 (ii) Find the atoms.  
 (iii) Is L complemented?  
 (iv) Is L distributive?

*(a, 2, 3) 1/2*  
*(a, x, y) 1/2*  
*(a, 10, 12) 1/2*  
*(2, 1) 1/2*

**SECTION-C**

Q6 (a) If  $u = \sin^{-1} \left\{ \frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}} \right\}^{1/2}$  then show that

(i)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{-1}{12} \tan u$

(ii)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$  (6.5)

(b) Find the equations of the spheres through the circle  $x^2 + y^2 + z^2 = 5$ ,  $x + 2y + 3z = 3$  and touching the plane  $4x + 3y = 15$ . (6)

Q7 (a) Find the maxima and minima of the function  $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$ . (6.5)

(b) Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $4x - 3y + 1 = 0 = 5x - 3z + 2$  are coplanar. Also find their point of intersection. (6)

**SECTION-D**

Q8 By changing to polar co-ordinates, evaluate  $\iint_R (x^2 + y^2) dx dy$  where R is the region in xy-plane bounded by  $x^2 + y^2 = 4$  and  $x^2 + y^2 = 9$ . (12.5)

Q9 Find the volume of the sphere  $x^2 + y^2 + z^2 = a^2$ . (12.5)

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$\frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} \Rightarrow -\frac{1}{6}$

$\frac{1}{2} - 1$