MSI-COPY

(Please write your Exam Roll No.) Exam Roll No. ..

END TERM EXAMINATION

SECOND SEMESTER [BCA]-MAY 2007

Paper Code :BCA-102 Subject: Mathematics-II (2005)

Time : 3 Hours Maximum Marks : 75

Note: Q.1 is compulsory and carries 25 marks. Attempt four questions selecting one from each Unit.

Q1. (a) If $A \subseteq C$ and $B \subseteq D$, then show that $AXB \subseteq CXD$ (2)

(b) Determine whether the relation R on the set A is an equivalence relation. A is the set of positive numbers, and a relation R is defined as aRb if $a=b^K$, $(a=b^K)$ where k is some positive integer. (1.5)

Not.

(c) R be a reflexive relation on a set A. Show that R is an equivalence relation if and only if (a,b) and $(a,c) \in R$ implies that $(b,c) \in R$. (2)

i.e. (a,b) & $(a,c) \in R \Rightarrow (b,c) \in R$

(d) Let C denote the set of complex numbers and let R denote the set of real numbers. Prove that the mapping $f: C \to R$ given by $f(x+iy)=|x+iy|$, where x and y are real ,is neither one to one nor onto. (3)

(e) Find the distance of the point (1,-2,3) from the plane $x-y+z=5$ measured parallel to the line (2)

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$$

(f) If the three thermodynamic variables P,V,T are connected by a relation $f(P,V,T)=0$, show that (3)

$$\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$

(g) Show, by double integration, the area between the parabolas $y^2=4ax$ & $x^2=4ay$ is $\frac{16}{3}a^2$. (2.5)

(h) Let S be any non empty set and P(S) be the power set of S. If '$\subseteq$' (a supset of) is a relation defined on P(S), then show that $(P(S),\subseteq)$ is a poset. (3)

(i) If Z is a homogeneous function of degree n in x &y, show that (3)

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial z^2}=n(n-1)z$$

(j) For any set A and B, answer the following question: (3)

(a) Is the set $AX\phi$ well defined? yes

(b) If $AXB=\phi$ what can you say about the sets A and B? $A=\phi, B=\phi$

(c) Is it possible that $AXA=\phi$, for some set A? only if $A=\phi$.

UNIT-I

Q2 (a) (i) Show that the transitive closure of a symmetric relation is symmetric. (3)

(ii) Let R be a transitive and reflexive relation on A. Let T be a relation on A such that (a,b) is in T if and only if both (a,b) and (b,a) are in R. Show that T is an equivalence relation. (3.5)

(b) Let R be a binary relation and $S=\{(a,b)|(a,c)$ & $(c,b) \in R$, for some C}.Show that if R is an equivalence relation then S is also an equivalence relation. (6)

OR

Q3 (a) Prove that if R is reflexive and transitive, then $R^n = R$ for all n.

(b) A survey was conducted among 1000 people. 595 of them are democrats. 595 wear glasses and 550 like ice cream. 395 democrats wear glasses. 350 democrats like ice cream. 400 of the people wear glasses and like ice cream. 250 democrats wear glasses and like ice cream.

Answer the following questions:

(i) How many people are not democrats who do not wear glasses and do not like ice cream. (4)

(ii) How many people are democrats who do not like ice cream and donot wear glasses.

(c) Let R and S be binary relations from A to B. Is it true that $(R\cup S)^{-1} = R^{-1} \cup S^{-1}$. Justify your answer. (3.5)

UNIT-II

Q4. (a) What can you say about the relation R on set A if R is a partial order and an equivalence relation? (3.5)

(b) Determine whether D_n is a finite Boolean algebra, where (i) n=12 (ii) n=40 (iii) n=75 (iv) n=21 (v) n=70? (5)

(c) Find all the maximal, minimal elements and greatest and least elements (if exist) of the Poset (A, ≤), A ={2,3,4,6,8,24,48} and ≤ is defined as the partial order of divisibility. (4)

OR

Q5. (a) Let A {1,2,3,5,6,10,15,30} be a set and R be a relation of divisibility on A. Show that R is a Partial order on A and draw a Harse diagram of R. Is R a linear order relation? What about if A= {2,4,8,16,32}? (6.5)

(b) Let L be a bounded lattice with at least two elements. Show that no element of L is its own complement? (3)

(c) Find the complement of each element in D_{42}. (3)

UNIT-III

Q6. (a) Show that the plane ax + by + cz + d = 0 touches the surface $px^2 + qy^2 + 2z = 0$ if $\frac{a^2}{p} + \frac{b^2}{q} + 2cd = 0$ (6)

(b) If f(x,y)=0 show that (6.5)

$$\left(\frac{\partial f}{\partial y}\right)^3 \frac{d^2 y}{dx^2} = 2\left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right)\left(\frac{\partial^2 f}{\partial x \partial y}\right) - \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial x}\right)^2 \left(\frac{\partial^2 f}{\partial y^2}\right)$$

OR

Q7. (a) In a plane triangle, Find the maximum value of CosA CosB CosC? (4)

(b) Transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates. (4)

(c) The equation $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial t^2}$ refers to the conduction of heat along a bar without radiation. Show that it $u = Ae^{-gx} \sin(nt - gx)$ where A,g, n are positive constants then $g = \sqrt{\frac{n}{2\mu}}$ (4.5)

UNIT –IV

Q8. (a) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ the cylinder $x^2 + y^2 = 2ay$ and a plane z = 0. (6.5)

(b) Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2 + r^2}}$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$ (6)

OR

Q9 (a) Evaluate $\iint r \sin\theta \, dr\, d\theta$ over the cardiode $r = a(1 - \cos\theta)$ above the initial line. (6.5)

(b) Calculate the area included between the curve $r = a(\sec\theta + \cos\theta)$ and its asymptate. (6)

****