An Overview of Quantum Inspired Evolutionary Algorithm

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ABSTRACT
Quantum Inspired Evolutionary Algorithm (QEA) is an evolutionary algorithm based on the concept of Quantum Computing. Quantum Computing makes use of quantum mechanical formula. In a quantum computer, the data to be manipulated, represented in quantum bits, exists in all possible states simultaneously, in superposition. This allows a single operation to operate over all these states at once. Like other evolutionary Algorithms, QEA is characterized by the representation of the individual, the evaluation function, and the population dynamics, but performs far better than evolutionary algorithms. QEA uses a string of Q-bits as a representation for individuals and a Q-gate as a variation operator to drive the individuals toward better solutions.

KEYWORDS
Quantum computing, Evolutionary algorithm.

1. INTRODUCTION
Quantum computation is the field that investigates the computational power and other properties of computers based on quantum-mechanical principles such as superposition and entanglement. A quantum computer uses quantum bits (qubits) for operation. A qubit is a bit of information that can be both zero and one simultaneously (superposition state). Two quantum particles both in superposition states can become entangled. This essentially means the extra information that each particle contains is linked with the other one. A quantum system uses these entangled particles to perform parallel computations. Quantum Computation is based on three axioms: Superposition axiom that tells us the possible states of a given quantum system, Measurement axiom that governs how much information about the state we can access and Unitary Evolution axiom that governs the evolution of state of a quantum system with time. Quantum circuit consists of wires and gates. They are just like classical circuits except that the gates involved are quantum gates. Wires carry qubit of information and Quantum gates are the applications of unitary transformation. Systems exhibiting Quantum Computing are “Exponentially Powerful”. Example: A system of 500 particles has \(2^{500}\) “Computing Power” [1][6][4][8]. Evolutionary Algorithms (EA) operate on a population of potential solutions, applying the principle of “survival of the fittest” to produce successively better approximations to a solution. At each generation of the EA, a new set of approximations is created by a process of selecting individuals according to their level of fitness in the problem domain and reproducing them using variation operators. This process may lead to the evolution of population of individuals that are better suited to their environment than the individuals from which they were created, just as in natural adaptation. Concept of quantum computing is proposed in evolutionary algorithm to represent the individuals effectively, to explore the search space with a smaller number of individuals and to exploit the search space for a global solution within a short span of time. Quantum Evolutionary Algorithm (QEA) uses a Q-bit as a probabilistic representation. QEA is not a quantum algorithm, but a novel evolutionary algorithm for a classical computer which provides fast speed up to Evolutionary algorithms. [2][5].

2. SCOPE
Quantum computers can vastly increase the speed of computation of the normal digital computers because of which they are used for solving NP complete Problems and helps in breaking most modern public-key cryptosystems. Computers used for personal purposes may not need the speed and performance delivered by quantum computers because of their limited scope of usage. However, quantum computers can prove beneficial for industries or government where high levels of data encryption may be required for secure data communication over the internet. Also, because of the extremely fast speed, quantum computers may find their usage in database search where millions of profiles can be searched for a keyword almost instantaneously and also in solving optimization problems that require exponential running time [3].

3. QUANTUM INSPIRED EVOLUTIONARY ALGORITHM
The QGA is a probabilistic algorithm similar to other evolutionary algorithms. It is characterized by principles of quantum computing including qubits and probability amplitude. It uses a qubit representation instead of the usual binary, numeric, or symbolic representations [7]. Concept of quantum computing is proposed in evolutionary algorithms to represent the individuals effectively, to explore the search space with a smaller number of individuals and to exploit the search space for a global solution within a short span of time [5].

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This journal is cited as: JIMS 8i-Int’l J. of Inf. Comm. & Computing Technology(IJICCT)

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QEA maintains a population of Q-bit individuals, 
\[
Q(t) = \{q_1, q_2, ..., q_n\}
\]
at generation \( t \), where \( n \) is the size of population. Each individual \( q_j \), is represented by a chromosome of \( m \) genes where each gene comprise of \( k \) qubits.

For simplicity, let us assume that one gene comprises of one qubit (\( k = 1 \)). Representation of binary quantum chromosome depicting each individual is as follows:

\[
q_j = \begin{bmatrix}
\alpha_{j1}^i & \alpha_{j2}^i & \cdots & \alpha_{jm}^i \\
\beta_{j1}^i & \beta_{j2}^i & \cdots & \beta_{jm}^i
\end{bmatrix}
\]

where a qubit is a two state quantum system represented by a single electron that is in one of two states denoted by \( |0\rangle \) and \( |1\rangle \), or a superposition of the two. The state of a qubit can be written in Dirac notation as: \( |\Psi\rangle = \alpha |0\rangle + \beta |1\rangle \), where \( \alpha \) and \( \beta \in \mathbb{C} \) (Complex Vector Space).

Also, \( |\alpha|^2 + |\beta|^2 = 1 \). Here, \( |\alpha|^2 \) and \( |\beta|^2 \) denotes the probability of system being in state \( |0\rangle \) and \( |1\rangle \) respectively.

Above chromosome is simply a string of \( m \) qubits i.e. \( m \) is the string length of the Q-bit individual. Here \( j = 1, 2, ..., n \) represents \( n \) individuals. Hence, each individual represents \( 2^m \) values simultaneously. When the measurement is done in a quantum system, superposition disappears and only one value becomes available for use i.e. qubit chromosome converges to a single state and the diversity given by the superposition of states disappears gradually. QEA being only quantum inspired and not a true quantum algorithm, assumes that superposition state remains intact [7].

4. QEA ALGORITHM

QEA procedure as described in [5] is reproduced below:

**Procedure QEA:**

\[
\begin{align*}
\text{begin} \\
& t < 0 \\
& \text{i. initialize } Q(t) \\
& \text{ii. determine } P(t) \text{ by measuring of } Q(t) \\
& \text{iii. evaluate } P(t) \\
& \text{iv. store the best solutions among } P(t) \text{ into } B(t) \\
& \text{v. while (not termination condition) do begin} \\
& \quad t < t + 1 \\
& \quad \text{vi. determine } P(t) \text{ by measuring } Q(t + 1) \\
& \quad \text{vii. evaluate } P(t) \\
& \quad \text{viii. modify } Q(t) \text{ using Q-gates} \\
& \quad \text{ix. store the best solutions among } B(t - 1) \text{ and } P(t) \text{ into } B(t) \\
& \quad \text{x. store the best solution } b \text{ among } B(t) \\
& \text{end}
\end{align*}
\]

**Stepwise Explanation of procedure QEA:**

i. In the step „initialize \( Q(t) \), \( \alpha_i^0 \) and \( \beta_i^0 \), \( i = 1, 2, ..., m \) in initial generation for all \( n \) individuals are initialized with \( 1/ \sqrt{2} \). It means that each individual, \( q_j^0 \), \( j = 1, 2, ..., n \) represents the linear superposition of all the possible states with the same probability.

ii. This step makes binary solutions in \( P(0) \) by measuring the states of \( Q(0) \), where 

\[
P(0) = \{x_1^0, x_2^0, ..., x_n^0\}
\]

at generation \( t = 0 \).

iii. Each binary solution \( x_j^0 \) is evaluated to give a measure of its fitness.

iv. The initial best solutions are then selected among the binary solutions \( P(0) \), and stored into \( B(0) \), where

\[
B(0) = \{b_1, b_2, ..., b_n\}
\]

and \( b_j = b_j \) at \( t = 0 \) is the same as \( x_j^0 \) at the initial generation.

v. Until the termination condition is satisfied, QEA is running in the while loop.

vi,vii In the while loop, binary solutions in \( P(t) \) are formed by observing the states of \( Q(t - 1) \) as in step ii, and each binary solution is evaluated for the fitness value.

viii. In this step, Q-bit individuals in \( Q(t) \) are modified by applying Q-gates defined below.

\[
U(\Delta \theta) = \begin{bmatrix}
\cos (\Delta \theta) & -\sin (\Delta \theta) \\
\sin (\Delta \theta) & \cos (\Delta \theta)
\end{bmatrix}
\]

where \( \Delta \theta \), \( i = 1, 2, ..., m \) is a rotation angle of each Q-bit toward either 0 or 1 state depending on its sign. \( \Delta \theta \) should be designed in compliance with the application problem. \( \Delta \theta \) can be obtained from following lookup table as a function of the i-th bit of the best solution \( b_i \), the i-th bit of the binary solution \( x_i \), and some meaningful conditions. The magnitude of \( \Delta \theta \) has an effect on the speed of convergence, but if it is too big, the solutions may diverge or converge prematurely to a local optimum. If the objective function is f, \( \Delta \theta \) may be determined as mentioned in Fig 1 [2].

The best solutions among \( B(t - 1) \) and \( P(t) \) are selected and stored into \( B(t) \), and if the best solution stored in \( B(t) \) is better fitted than the stored best solution \( b \), the stored solution \( b \) is replaced by the new one [2].
An Overview of Quantum Inspired Evolutionary Algorithm

<table>
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<th>$x_i$</th>
<th>$b_1$</th>
<th>$f(x)$</th>
<th>$f(b)$</th>
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</table>

Fig 1: Lookup table [2]

5. CONCLUSION
Quantum computation is the field that investigates the computational power and other properties of computers based on quantum-mechanical principles such as superposition and entanglement.
Quantum Inspired Evolutionary Algorithm (QEA) is inspired by the concept of quantum computing. The QEA is characterized by the Q-bit representation for the population diversity, the observation process for producing a binary string from the Q-bit individual, the update process for driving the individuals toward better solutions by the Q-gate and the termination condition which can be given by the probability of the best solution.

6. FUTURE SCOPE
QEA has been applied to several NP-hard problems such as 0-1 knapsack problem, travelling salesman problem etc. We intend to apply it to other NP hard problems to take advantage of quantum phenomena to efficiently speed up classical computation.

REFERENCES