(Please write your Exam Roll No.) *Exam Roll No.*

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END TERM EXAMINATION

FIRST SEMESTER [MCA] DECEMBER-2008

Paper Code: MCA 107	**Subject: Discrete Mathematics**
Paper Id: 44107	**(Batch: 2004-2008)**

Time: 3 Hours **Maximum Marks: 60**

Note: Attempt any five questions. Q. 1 is compulsory.

Q.1 (a) If f_A is characteristic function defined on set A f_B is characteristic function defined on set B. Show that $f_{AuB} = f_A + f_B - f_A f_B$
(1.5)

(b) Give the n^{th} term of the sequence 1,2,4,16,…..
(0.5)

(c) Prove that A - (A - B) $\subseteq$ B ; A,B are sets
(1.5)

(d) What is the number of distinct four cards hands which can be dealt from a deck of 52 cards?
(1.5)

(e) Prove by induction $n < 2^n$ (n>1)
(1.5)

(f) Let 2^+ be set of positive integers, show that (2^+ , $\leq$) is a partial ordered set.
(1.5)

(g) Define a Lattice.
(1)

(h) In a Boolean Algebra (L , $\leq$) prove that (a')' = a.
(1)

(i) Solve the Boolean Expression (x $\wedge$ y $\wedge$ z') V (x $\wedge$ y' $\wedge$ z') using K-map.
(0.5)

(j) Is the argument valid (do not use the truth table)? If prices are lowered my savings increase –

$\therefore$ my savings increase

prices are lowered

(1.5)

(K) Let (G .) be a group and let x, y $\in$ G. Show that $(xy)^{-1} = y^{-1}x^{-1}$

(1.5)

(l) Give an example of group homomorphism.

(1)

(m) Give an example of Hamiltonian Path.

(1)

(n) In a connected graph give the relationship between the vertices and edges of the graph, if any.

(1)

(o) Define context free grammar.

(1)

(p) For the following State Transition Table

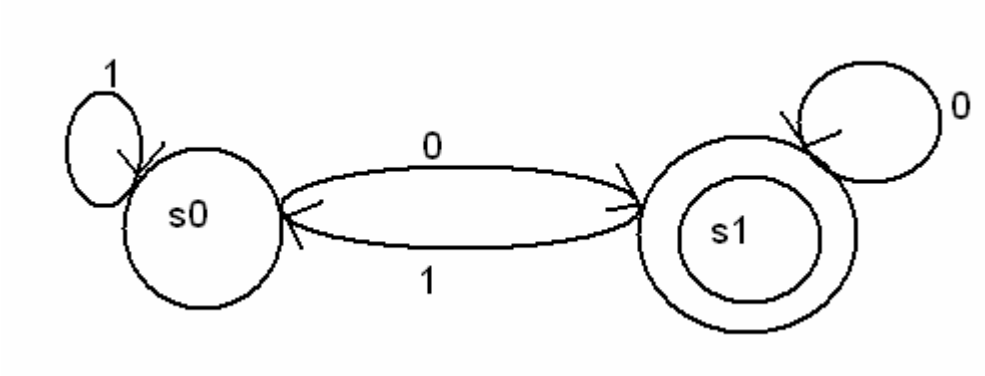
(1)

	a	b
S_0	S_0	S_1
S_1	S_2	S_0
S_2	S_1	S_2

Where S_i 's are states and a, b are inputs. Draw the labelled diagraph for it.

(q) Describe the language recognized by the machine.

(1.5)



Q2. (a) Show that for propositions p and v p<->q and (p $\wedge$ q) v (-p $\wedge$ -q) are logically equivalent.
(3)

(b) What do you mean by indirect proof? Using indirect proof prove that

"If 5n+3 is even if n is odd".?
(3)

(c) Describe an efficient technique for representing a finite set in computer.
(4)

Q3. (a) Let A be set of positive integers. Let R be a relation on A defined as

(a, b) $\in$ R $\Leftrightarrow$ (a-b) is divided by m $\neq$ 0, where m is a positive integer. Show that r is an equivalence relation.
(3)

(b) Find the generating function for finite sequence 1, 4, 16, 64, 256.
(2)

(c) State and prove the principal of Inclusion and Exclusion.
(2)

Q4. (a) Draw the Hasse diagram for the set of all positive divisors of 30. Applying topological sort indicate the steps of sorting the elements using Hasse diagram.
(5)

(b) Let L be a bounded distributed lattice. Show that if the complant of an element in l exists then it is unique.
(5)

Q5. (a) Find sum of product expression for the following using K-map

F(A, B, C, D) = ABCD + AB$\overline{C}$D + AB$\overline{CD}$ + A$\overline{B}$C$\overline{D}$ + A$\overline{B}\overline{C}$D
(5)

Don't care d(A, B, C, D) = $A\overline{B}CD + \overline{A}\overline{B}\overline{CD} + \overline{A}B\overline{CD} + A\overline{B}\overline{CD}$

(b) What do you mean by proposition, propositional variable and compound statements? Give an example of each
(5)

Q6. (a) Let G be a connected undirected graph. Prove that there is a simple path between every pair of vertices in G. How will you find the number of paths of length m(>0) in G? Prove it.
(3+3)

(b) Write the steps for constructing Euler circuits in a connected multigraph.
(4)

Q7. (a) Let G be a group with identity e. Show that if $a^2 = e \ \forall \ a \in G$, then G is abelian.
(3)

(b) Let G be a group of integers under addition operation.
Let H ={2m/m $\in$ z }. Show that H is a subgroup of G.
(3)

(c) Let (Gi) be a finite group and (Hi) be a subgroup of (Gi). How will you obtain the cosets of H in G? Explain.
(4)

Q8. (a) Let V = {S, A, B, a, b}, T = {a, b} symbols of usual meaning. Find the language generated by the grammar (V, T, S, P) where the set of production rules P is S -> AA, A -> aAb, B -> bBa, A -> λ, B -> λ.
(5)

(b) Produce a FSM that adds two binary numbers.
(5)

Q9. (a) Define a finite state machine. What are the different types of FSM. (3)

(b) Give an example of context sensitive grammar. (3)

(c) Give phrase structure grammar to generate the set $\{0^m 1^n \mid m, n$ are non negative integers$\}$. (4)

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